AAT Name _____

2-5 Exponential Models Date _____ **#9**

Goal: Construct mathematical models of situations which are exponential in nature.

Warm Up: Hafiz keeps track of how much money he has. In one fixed annuity investment initiated 10 years ago, he had $218,201 three years ago and he has $256,221 now.

a. Use your ExpReg function to calculate the annual yield from his investment.

b. How much was the initial investment?

Finding an Exponential Function Using a System of Equations

Example 1: Now, let's do the above situation by hand.

a. Find an exponential model for the data.

b. Predict how much he would have in 3 years.

Example 2: Huntley, Illinois has been a small farming town. But when a large housing development was built, the population growth pattern changed. Two special censuses gave village planners the data in the table at the right.

Year	Population
2003	12,270
2005	16,719

Source: The Village of Huntley

a. Find an exponential model for the data. Let $p(t)$ be the population t years after 2000.

b. Predict what the population would haven been in 2016.

Questions

Questions	**Exponential Regression**

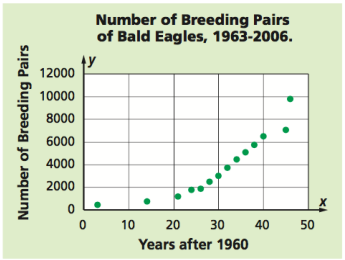
Example 3: Bald eagles were once threatened with extinction. In the 48 contiguous states, their numbers were at an all-time low of 417 in 1963. But protection programs helped them rebound. In 2007, they were removed from the list of endangered species kept by the U.S. Fish and Wildlife Services.

Year	Number of Breeding Pairs
1963	417
1974	791
1981	1188
1984	1757
1986	1875
1988	2475
1990	3035
1992	3749
1994	4449
1996	5094
1998	5748
2000	6471
2005	7066
2006	9789

Source: U.S. Fish and Wildlife Services

a. Use a statistics utility to fit an exponential model of the form $f(x) = ab^x$ to the data. Let x be "years after 1960." Report the values of a and b in the table at the right to the nearest thousandth.

b. Superimpose the graph of the exponential model on the scatterplot in your calculator and describe how well the exponential curve fits the data.



c. Identify the initial amount and the growth factor and explain their meanings.

d. Find the residuals for the model's predicted values for 2000 and 2005.

e. Extrapolate the population for today.

Example 4: The table below contains breaking strength for a multi-strand steel cable.

Diameter (mm)	5	8	12	14	16	22	30	36	40	48
Breaking Strength (kg)	355	755	1,720	2,085	2,535	4,705	8,795	12,430	14,520	21,270

a. Using the entire data set and a graphing calculator, determine the model for the data.

b. Identify the initial breaking strength and the growth factor and explain their meanings.

c. Use the model to estimate the breaking strength of a 44-mm diameter multi-strand cable. Is your estimate consistent with the data?

Half Life and Exponential Decay

Radioactive elements are useful in situations involving detective work, such as diagnosing health problems with barium x-rays or finding the age of archeological artifacts with carbon dating.

The **half-life** of a radioactive element is the amount of time it takes an original quantity to decay to half that amount. If you know the half-life of a radioactive element and the amount of the substance at one point in time, you can find the original amount.

In 2007, the element polonium was in the news when London police detectives investigated the poisoning of former Russian KGB agent Alexander Litvinenko. Since polonium had never been known to be used in a poisoning, the authorities did not look for evidence of it until weeks after the crime had taken place. As a consequence, they had to work backwards from the evidence to calculate the amount of polonium used on the victim. They made use of the fact that the half-life of polonium is 138 days.

Questions

Questions	Example 3: Detectives in the Litvinenko investigation found polonium on a cup in a hotel that he had visited. Suppose that 4 micrograms were found, and it had been 30 days since Litvinenko was there. a. Find how much polonium was on the cup originally. b. Derive a model for this situation. Example 4: A certain substance has a half-life of 24 years. If a sample of 80 grams is being observed, how much will remain in 50 years? When will only 5 grams remain?

Summary: